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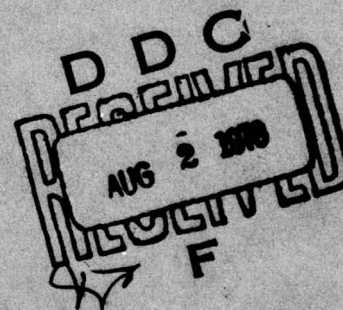
The Behavior of Gridded Spherical and Planar Electron Probes in a Non-Maxwellian Plasma

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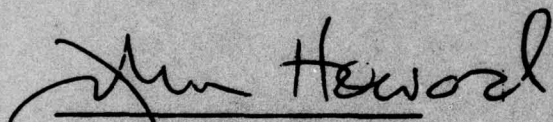HANSCOM AFB, MASSACHUSETTS 01731

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
The responses of gridded spherical and planar electrostatic probes to plasmas with "kappa" type distribution functions are analyzed along lines developed by Mott-Smith and Langmuir. Specifically, we consider a case that could be encountered when a satellite is in the outer reaches of the plasmasphere. Practical rules are developed for calculating the plasma's spectral index (K), its mean thermal energy, and density. It is shown that treating the plasma as though it were Maxwellian, can lead to serious overestimates of the mean thermal energy but less serious overestimates of the electron density.

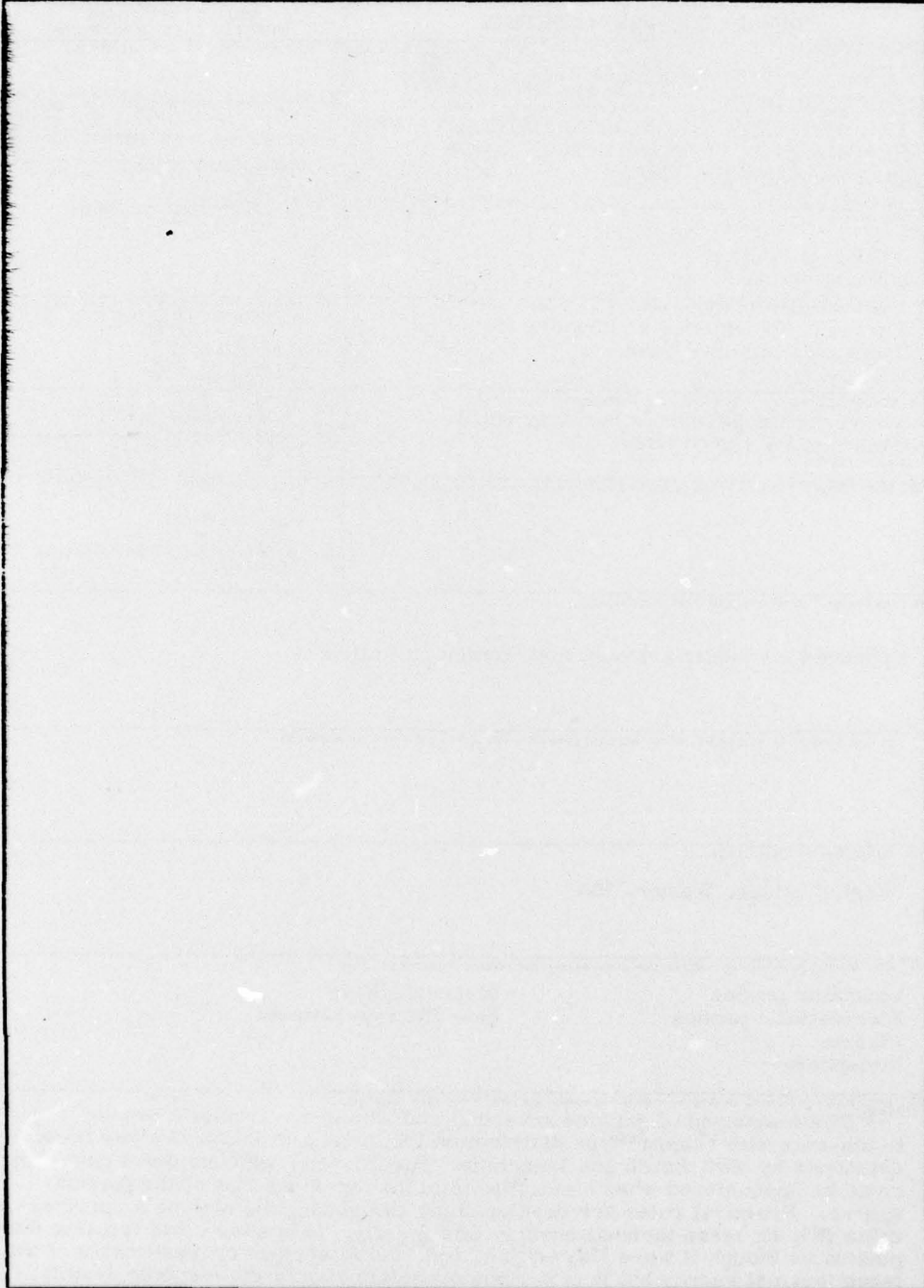DD FORM 1473 1 JAN 73   EDITION OF 1 NOV 65 IS OBSOLETE

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Contents

Illustrations

The Behavior of Gridded Spherical and Planar Electron Probes in a Non-Maxwellian Plasma

1. CURRENT CALCULATIONS

Since the pioneering efforts of Mott-Smith and Langmuir,[1] many investigations have analyzed the response of electrostatic probes in stationary plasmas.[2] Because of their sensitivity to low energy particles (<1 eV), electrostatic probes of various geometric configurations have been used to measure ionospheric parameters. In the coming year, several experiments aboard the SCATHA satellite will attempt to measure the thermal properties of plasmas at $\sim 6R_E$ above the equator. At such great altitudes, it is possible that the distribution function of thermal electrons varies from the Maxwellian distributions commonly found in the lower ionosphere. In anticipation of such a variation, we here consider the responses of gridded spherical and planar electrostatic analyzers to a type of non-Maxwellian plasma already encountered in several regions of space. The probes that we consider here are made up of either a collecting sphere surrounded by a concentric wire mesh grid (Figure 1a), or a planar collector with circular gridded aperture (Figure 1b). The collectors are held at sufficiently high positive potentials to

1. Mott-Smith, H. M., and Langmuir, I. (1926) The theory of collectors in gaseous discharges, Phys. Rev. 28:727-763.

2. Swift, J. D., and Schwar, M. J. R. (1969) Electrical Probes for Plasma Diagnostics, Amer. Elsevier, New York.
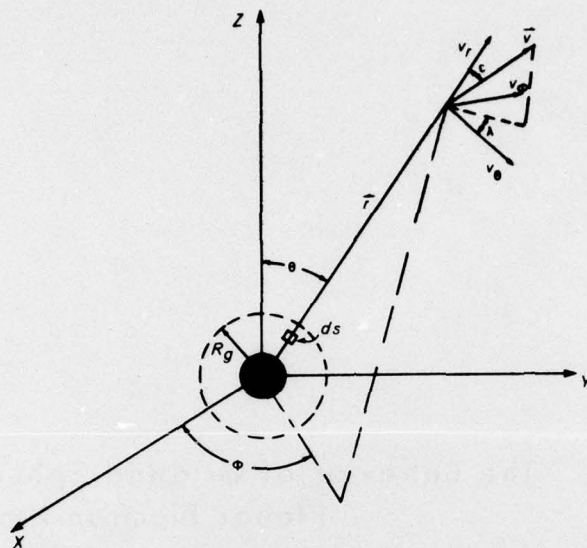
Figure 1a. Velocity of a Particle Relative to a Gridded Spherical Electrostatic Analyzer of Radius R_G. (θ, ϕ) and (ϵ, λ) are the colatitude and azimuth angles in configuration and velocity spaces, respectively
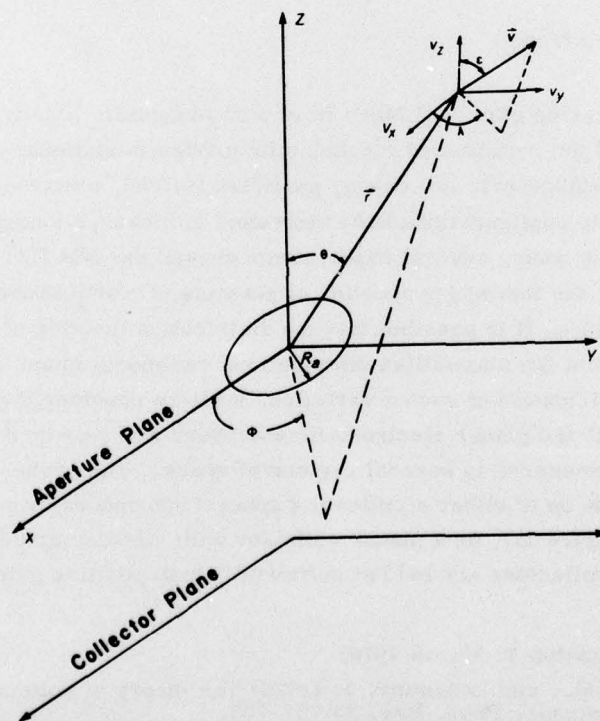
Figure 1b. Velocity of a Particle Relative to the Circular, Gridded Aperture of a Planar Electrostatic Analyzer. The aperture radius is R_A. (θ, ϕ) and (ϵ, λ) are the colatitude and azimuth angles in configuration and velocity spaces, respectively

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screen ambient thermal ions from detection. The grids may be either swept or stepped in voltage. Similar instruments have been flown on several Air Force satellites. Equations appropriate for spherical probes are developed explicitly in the text and those for planar geometries in Appendix A.

The current (I) measured by a gridded spherical electrostatic probe with an applied retarding voltage (V) due to a Maxwellian electron gas of temperature (T) is[3]

$$I(V) = \pi R_G^2 \, \beta \, n \, q \, \bar{c} \, e^{-\frac{qV}{kT}} \tag{1}$$

where: β and R_G are the transmission coefficient and radius of the grids; n, q, and $\bar{c}$ are the electron density, charge and mean thermal speed $(8kT/\pi m)^{1/2}$, respectively; and m is the mass of an electron and k is the Boltzmann constant. T is calculated by examining the current characteristic (ln(-I) versus V curve) for a linear portion and using the relationship

$$T = -\frac{q}{k} \left[d \ln (-I)/dV \right]^{-1} . \tag{2}$$

In the analysis of electrostatic probe measurements taken in the ionosphere, it is commonly assumed that the plasma is Maxwellian. In the lower reaches of the ionosphere where collision times are short in comparison to the measurement time, the Maxwellian assumption is justified by the H-theorem. In the topside ionosphere, the assumption is justified by experience. Frequently, however, plasmas found in deep space have long relaxation times. In such cases, a Maxwellian interpretation may be invalid. For example, the electrons measured in the plasma sheet[4] and magnetosheath[5] have "kappa-type" distribution functions of

$$f_K(v) = \frac{n}{w^3} \, \frac{\Gamma(K+1)}{(\pi K)^{3/2} \, \Gamma(K-1/2)} \, \frac{1}{\left[1 + \frac{v^2}{Kw^2} \right]^{K+1}} \tag{3}$$

3. Smiddy, M., and Stuart, R.D. (1969) An Analysis of the Behavior of a Multi-grid Spherical Sensor in a Drifting Maxwellian Plasma, AFCRL-69-0013.

4. Vasyliunas, V.M. (1968) A survey of low-energy electrons in the evening sector of the magnetosphere with OGO 1 and OGO 3, J. Geophys. Res. 73:2839-2884.

5. Formisano, V., Moreno, G., Palmiotto, F., and Hedgecock, P.C. (1973) Solar wind interaction with the earth's magnetic field, 1, magnetosheath, J. Geophys. Res. 78:3714-3730.

where w is the most probable thermal speed and K is the slope of the directional differential flux at high energies.[4] In the limit $K \to \infty$, this distribution function reduces to the Maxwellian case. We now calculate the current measured by a spherical probe given by electrons with a kappa distribution.

The current to a sphere of radius r is

$$I = q \iint_{S \quad \vec{v}} f(v)\, \vec{v} \cdot \hat{n}\, dS\, d^3 v \tag{4}$$

where $\hat{n}$ is an inward directed unit surface-normal vector. The surface element $dS = r^2 \sin \theta\, d\theta\, d\phi$ and the velocity element $d^3 v = v^2\, dv\, \sin \varepsilon\, d\varepsilon\, d\lambda$. The angles (θ, ϕ) and (ε, λ) are the colatitude and azimuth in configuration velocity spaces (Figure 1) respectively. In calculating I, the velocity space integration is done only over those orbits which penetrate the grid.[1,3] From the conservation of energy and angular momentum, the integration limits for v and ε are $0 \leq v_R \leq v < \infty$ and $0 \leq \varepsilon \leq \varepsilon_R$ where

$$v_R = (2\, qV/m)^{1/2}$$

and

$$\varepsilon_R = \sin^{-1}\left[\frac{R_G}{r} \sqrt{1 - \frac{v_R^2}{v^2}} \right]$$

where V is the retarding potential of the grid. Assuming that the electrons are isotropically distributed, integration over configuration and velocity space angles gives

$$I = 4\pi^2\, R_G^2\, q \int_{v_R}^{\infty} \left(1 - \frac{v_r^2}{v^2}\right) f(v)\, v^3\, dv \quad . \tag{4'}$$

Using the distribution function given in Eq. (4) and allowing for the grid transmission gives

$$I_K(V) = 2\sqrt{\pi}\, \beta\, R_G^2\, nqw\, \frac{K^{1/2}\, \Gamma(K+1)}{\Gamma(K-1/2)} \left(1 + \frac{qV}{KE_o}\right)^{1-K} \tag{5}$$

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where $E_o = 1/2\, m\, w^2$ is the most probable thermal energy. We note that in the limit $K \to \infty$, Eq. (5) reduces to Eq. (1). An equation similar to (5) is derived in Appendix A for a planar sensor.

As a concrete example we have calculated the electron current to a gridded spherical electrostatic analyzer under the conditions $n = 100$ cm^{-3}, $E_o = 1$ eV, and $K = 3, 5, 10$. These conditions could be realized in the outer reaches of the plasmasphere in the equatorial plane. A short FORTRAN program for calculating $I_K(V)$ with $R_G = 3.18$ cm and $\beta = 0.546$ is given in Appendix B. Calculated values of log I as functions of applied voltage for the four values of K are given in Figure 2. For simplicity, we have assumed that the satellite ground is at plasma potential. Because the vehicle potential is constant during the retarding portion of an electron sensor sweep,[6] this assumption is not critical. In the Maxwellian case $(K = \infty)$, the log I - V curve is linear. The current characteristic, however, becomes more and more nonlinear as the value of K decreases. Slopes of the $K < \infty$ curves are less steep than in the Maxwellian case. Thus any attempt to analyze a $K < \infty$ characteristic as though the plasma were Maxwellian, leads to an overestimate of the mean thermal energy (c, Eq. (2)). The degree to which the density is miscalculated depends, in turn, on the degree to which E_o has been overestimated. It is thus useful to develop practical methods by which the parameters K, E_o, and n may be obtained from the current-voltage relation given in Eq. (5). We assume that a ln (-I)-V plot has a nonlinear shape indicating that $K < \infty$.

Equation (5) is of the form

$$I_K(V) = n\, E_o^{1/2}\, C_1\, F\,(K)\ \left(1 + \frac{qV}{KE_o}\right)^{1-K}$$

where $C_1 = \pi R_e^2\, \beta\, q\, (8\alpha/\pi m)^{1/2}$ and $F\,(K) = K^{1/2}\, \Gamma(K-1)/\Gamma(K-1/2)$. F (K) decreases monotonically from 1.3 to 1.0 as K increases from 3 to ∞. In the limit $qV/KE_o \gg 1$, the current is proportional to $(qV/KE_o)^{1-K}$. The term α in the definition of C_1 is 1.6×10^{-12} erg/eV. Thus,

$$K = 1 - \left. \frac{\partial \ln\,(-I)}{\partial \ln V} \right|_{\frac{qV}{KE_o} \gg 1} . \tag{6}$$

6. Smiddy, M., and Stuart, R.D. (1969) The Characteristics of a Space-Vehicle-borne Charged Particle Sensor, AFCRL-69-0519.
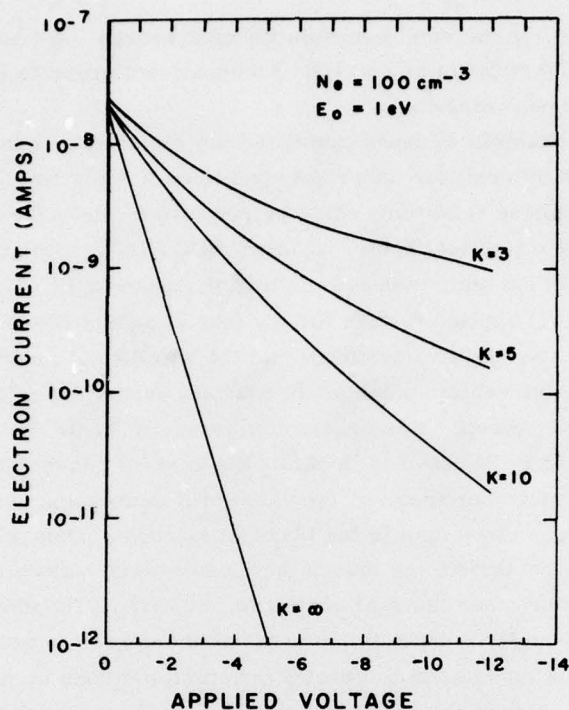
Figure 2. Semilog Plot of the Current-Voltage Curves to a Spherical Electrostatic Analyzer With R_G = 3.18 cm and β = 0.546 Due to a Plasma With an Electron Density of 100 cm^{-3}, a Most Probable Thermal Energy of 1 eV and Spectral Indices of 3, 5, 10 and ∞

Log (-I) versus log V plots for the cases studied above are given in Figure 3. We note that in the K = 3, 5 and 10 cases the log (-I) - log V curves decrease linearly at large retarding voltages, characteristic of a power law relationship.

Having determined the value of K from the slope of the log (-I) - log V curve at large values of qV/KE_o, we may now calculate E_o. In the region $V \to 0$ of Figure 2, the log (-I) - V curve approaches a linear relationship

$$\lim_{V \to 0} \frac{d \ln (-I)}{dV} = \left(\frac{1 - K}{K} \right) \frac{q}{E_o}$$

or
$$\tag{7}$$

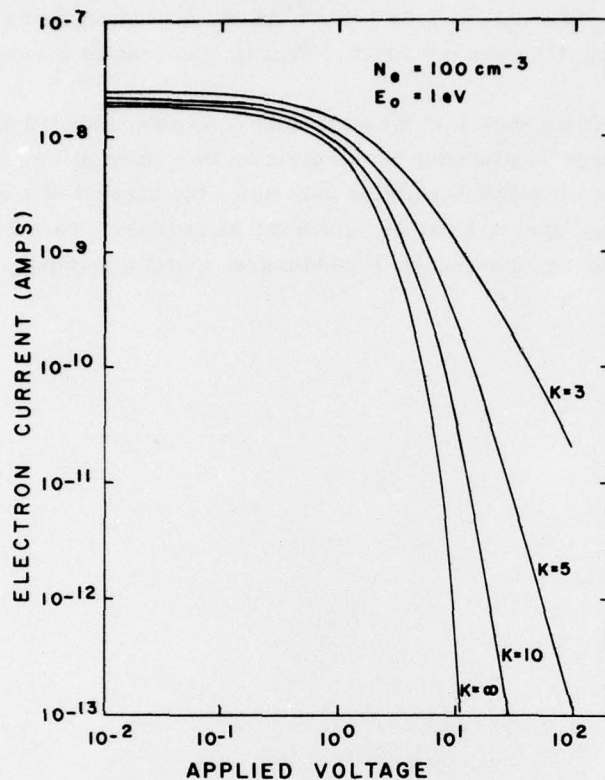$$E_o = \left(\frac{1 - K}{K} \right) q \left[\frac{d \ln (-I)}{dV} \right]^{-1} \Bigg|_{V=0}$$

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Figure 3.   Log-log Plot of Current-Voltage Curves
Under the Same Conditions Specified for Figure 2

In the Maxwellian approximation (Eq. (2)) $-q \, [d \ln (-I)/dV]^{-1} = kT$.   Thus if K = 3
the mean thermal energy is overestimated by 50 percent, and by 10 percent if
K = 10.

The electron density is calculated from the current measured when the sensor's grids are at plasma potential (V = 0)

$$n = \frac{I_K(0)}{C_1 \, F \, (K) \, E_o^{1/2}} \quad .$$

(8)

The error in the density calculation is not so severe as in the case of temperature.
Consider the case of K = 3, E_o = 1 eV and n = 100 given in Figure 2.   The current
at V = 0 is 2.42×10^{-8} A.   In the Maxwellian approximation F (K) = 1 and

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E_o = 1.5 eV. C_1 is equal to -1.86×10^{-10} for R_G = 3.18 cm and β = 0.546. Substitution in Eq. (8) gives n = 106.2. That is, the density is overestimated by 6.2 percent.

In Appendix A we show that for a circular-aperture, gridded planar probe the current-voltage relationship is the same as that given in Eq. (5). However, a spherical probe of radius R collects four times the current of a circular planar sensor of the same aperture radius, given the same plasma parameters. The above methods for determining K, E_o and n are, _mutatis mutandis_, directly applicable.

References

1. Mott-Smith, H.M., and Langmuir, I. (1926) The theory of collectors in gaseous discharges, Phys. Rev. 28:727-763.

2. Swift, J.D., and Schwar, M.J.R. (1969) Electrical Probes for Plasma Diagnostics, Amer. Elsevier, New York.

3. Smiddy, M., and Stuart, R.D. (1969) An Analysis of the Behavior of a Multigrid Spherical Sensor in a Drifting Maxwellian Plasma, AFCRL-69-0013.

4. Vasyliunas, V.M. (1968) A survey of low-energy electrons in the evening sector of the magnetosphere with OGO 1 and OGO 3, J. Geophys. Res. 73:2839-2884.

5. Formisano, V., Moreno, G., Palmiotoo, F., and Hedgecock, P.C. (1973) Solar wind interaction with the earth's magnetic field, 1, magnetosheath, J. Geophys. Res. 78:3714-3730.

6. Smiddy, M., and Stuart, R.D. (1969) The Characteristics of a Space-Vehicle-borne Charged Particle Sensor, AFCRL-69-0519.

The current across the gridded aperture of a planar sensor is

$$I = q \iint_{S} \int_{v_o} f(\vec{v}_o, \vec{r}_o) \, \vec{v}_o \cdot \hat{n} \, dS d^3 v_o \qquad \text{(A 1)}$$

where $\hat{n} = -\hat{z}$ is an inward directed surface normal unit vector (Figure 1b), $f(\vec{v}_o, \vec{r}_o)$ is the electrons' distribution function at the aperture. Thus $\vec{v}_o \cdot \hat{n} = -v_{zo}$. To solve equation (A 1) we assume that:

(1) at great distances from the aperture, the electrons have an isotropic, kappa distribution, and

(2) the retarding potential distribution is a function of z, the distance from the z = 0 plane, only.

These assumptions allow us to approximate the current to the sensor as the product of the aperture area (A) times the electron flux across the z = 0 plane.

For an electron to cross the z = 0 plane, at great distances from the plane it must have a sufficiently high velocity component in the z direction to overcome the potential barrier. The conditions are

$$v > v_R = (2qV/m)^{1/2}$$

and

$$0 \leq \mathrm{Sin}\ \varepsilon \leq \left(1 - \frac{v_R^2}{v^2}\right)^{1/2} = \varepsilon_R$$

where v and ε are the electron's speed and velocity space colatitude angle (Figure 1b) at a great distance from the detector. From the conservation of energy and momentum, it is easily shown that $v_{zo} d^3 v_o = v_z d^3 v$. Using the fact that the distribution function is a constant along particle trajectories (Liouville's Theorem), we can write

$$I = -q\ A \int_0^{2\pi} d\lambda \int_\pi^{\varepsilon_R} \mathrm{Cos}\ \varepsilon\ \mathrm{Sin}\ \varepsilon\ d\varepsilon \int_{v_R}^\infty f(v)\ v^3\ dv$$

Integration over velocity and configuration spaces give:

$$I = \pi\ A\ q \int_{v_R}^\infty \left(1 - \frac{v_R^2}{v^2}\right) f(v)\ v^3\ dv \quad . \tag{A2}$$

If we let $4\pi R_G^2$ be equal to the area of the "aperture" of a spherical sensor, then Eq. (A2) is identical to Eq. (4') in the text. Using the distribution function

$$f_K(v) = \frac{n}{w^3}\ \frac{\Gamma(K + 1)}{(\pi K)^{3/2}\ \Gamma(K - 1/2)}\ \frac{1}{\left(1 + \dfrac{v^2}{Kw^2}\right)^{K+1}}$$

gives

$$I_K(V) = 1/4\ \beta\ n\ q\ A\ \bar{c}\ \frac{K^{1/2}\ \Gamma(K - 1)}{\Gamma(K - 1/2)}\ \frac{1}{\left(1 + \dfrac{qV}{KE_o}\right)^{K+1}}$$

where $C = (8\ E_o/\pi m)^{1/2}$, and β is the transmission coefficient of the grid. Note that if we assume that the aperture is circular with a radius R_A, then $A = R_A^2$. Thus for grids of the same radius a spherical detector will collect four times more current than a circular planar sensor.

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Appendix B

```
C THIS PROGRAM IS DESIGNED TO CALCULATE THE CURRENT TO A SPHERICAL
C ELECTROSTATIC ANALYZER IF THE AMBIENT ELECTRONS HAVE A KAPPA
C DISTRIBUTION.   HE BIASES OF THE GRIDS ARE ASSUMED TO BE RETARDING
C FOR ELECTRONS. I.E. V<0.
C THE CURRENT EQUATION IS    CURV = 2.*DENS*Q*TRANS*(PG**2)*SQRT(PI)* WO*
C GAMMA(K-1)E*SQRT(K)/(GAMMA(K-.5)*(19+VOLT/RK*EO)**KMO)
C WHERE Q=1.6E-19COULOMBS, TRANS=.546 (TRANSMISSION FACTOR OF THE
C INSTRUMENT) AND VOLT IS THE APPLIED VOLTAGE OF THE GRID. PG IS
C THE RADIUS OF THE OUTER GRID (=3.18CM).   WO IS THE MEAN THERMAL VELOCITY.
C EO IS THE MEAN THERMAL ENERGY
      DIMENSION VOLT(13),ADD(13),DNM(13),CURV(13)
    1 READ(5,10) DENS,EO,K
   10 FORMAT(2F10.1,I5)
      PI=3.141593
      PI12=SQRT(PI)
      RK=K
      KMO=K-1
      EO12=SQRT(EO)
C CALCULATE  GAM1=GAMMA(K-1) = (K-2))
      RN= K-2.
      IF(RN) 7,7,8
    7 GAM1=1.
      GO TO 6
    8 GAM1=RN
    9 RN=RN-1.
      IF (RN) 6,6,5
    5 GAM1=GAM1*RN
      GO TO 9
    6 CONTINUE
```

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```
C CALCULATE GAM2 = GAMMA(K-1/2) = SQRT(PI)*(2K-3)))/2.**(K-1)
      L=3
      NUM=2*K-L
   18 L=L+2
      NUM1=2*K-L
      IF(NUM1-1)16,16,17
   17 NUM=NUM*NUM1
      GO TO 18
   16 GAM2=NUM*PI12/(2.**(K-1))
      FACT=GAM1*SQRT(RK)/GAM2
      WRITE(6,99)DENS,E0,RK
   99 FORMAT(3F11.1)
      DO 50 K=1,5
      DO 49 J=1,13
      GO TO (21,22,23,24,25) K
   21 VOLT(J)=(J-1)*0.001
      GO TO 30
   23 VOLT(J)=(J-1)*0.1
      GO TO 30
   22 VOLT(J)=(J-1)*0.01
      GO TO 30
   24 VOLT(J)=(J-1)*1.
      GO TO 30
   25 VOLT(J)=(J-1)*10.0
   30 ADD(J)=VOLT(J)/(RK*E0)
      DNM(J)=(1.+ADD(J))**KMO
      CURV(J)=1.95E-10*DENS*SQRT(E0)*FACT/DNM(J)

      WRITE(6,48)VOLT(J),CURV(J)
   48 FORMAT(2F20.5)
   49 CONTINUE
   50 CONTINUE
      GO TO 1
      END
```